

Entanglement-assisted local operations and classical communications conversion in the quantum critical systems

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Conversions between the ground states in quantum critical systems via entanglement-assisted local operations and classical communications (eLOCC) are studied. We propose a new method to reveal the different convertibility by local operations when a quantum phase transition occurs. We have studied the ground state local convertibility in the one dimensional transverse field Ising model, XY model and XXZ model. It is found that the eLOCC convertibility sudden changes at the phase transition points. In transverse field Ising model the eLOCC convertibility between the first excited state and the ground state are also distinct for different phases. The relation between the order of quantum phase transitions and the local convertibility is discussed.

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I. INTRODUCTION

The recent development in quantum information processing (QIP) [1] has provided much insight into the quantum many-body physics [2]. In particular, using the ideas from QIP to investigate quantum phase transitions has drawn vast attentions and has been successful in detecting a number of critical points of great interest. For instance, entanglements measured by concurrence, negativity, geometric entanglement and von Neumann entropy are studied in several critical systems [3–7]. It was found that the von Neumann entropy diverges logarithmically at the critical point [3], and the concurrence shows a maximum at the quantum critical points of transverse field Ising model and XY model [4]. The second order phase transition in the XY model can also be determined by the divergence of the concurrence derivative or the negativity derivative with respect to the external field parameter [5, 6]. Apart from entanglement, other concepts in quantum information such as mutual information and quantum discord which feature some singularities at critical points were also found to be useful in detecting quantum phase transitions [8, 9]. Recent studies in entanglement spectrum can be applied to the detection of quantum phase transitions [10, 11], too. At the same time, fidelity as well as the fidelity susceptibility of the ground state also works well in exploring numerous phase transition points in various critical systems [12–14]. These methods have achieved great success in understanding the deep nature of the different phase transitions, especially the structure of the correlations revolved [15, 16].

However, in the previous studies although the concepts from quantum information were investigated, they were not fully explored, because they were merely used as some common order parameters, and some important operational properties were missing. In this paper, from a new point of view, we reveal the operational properties of the critical systems by fully studying the ground state Rényi entropy and show that the operational property

sudden changes at the quantum phase transition point, so that we could put forward a new proposal to investigate the quantum phases and quantum phase transitions. To be specific, we investigate a many-body system whose Hamiltonian is $H(\lambda)$ with a variable parameter λ . The system has a critical point at $\lambda = \lambda_c$, which separates two quantum phases. We examine the possibility for the ground state of $H(\lambda)$ to convert into the ground state of $H(\lambda')$ by entanglement-assisted local operations and classical communications (eLOCC). If we can find a method to convert them, we say there is local convertibility between them, otherwise there is no local convertibility. Our motivation is that from the Rényi entropy interceptions of different states we can get the knowledge of their local convertibility, and this local convertibility is different for ground states from different phases. Thus, phase transitions can be distinct observed. Besides the operational meaning, our proposal has other advantages in that it is easy to moderate the accuracy, and the quantum phase transition points can be detected for small-sized system. In addition, we do not need a priori knowledge of the order parameters nor the symmetry. As we are revealing the local operation properties of different quantum phases, this paper is essentially concerned with deterministic conversion of a single copy of state, which would be different from the stochastic results [17] and asymptotic results [18].

The paper is organized as follows. In section II, we introduce some local conversions as well as their necessary and sufficient conditions, especially the eLOCC conversion, which is mainly focused on in this paper. In section III, we study the ground state local convertibility in the one dimensional spin half transverse field Ising model, XY model and XXZ model. In particular, for the transverse field Ising model we show how to determine the critical point numerically with small-sized systems by the finite size scaling analysis, and we also investigate the local convertibility between the ground state and the corresponding first excited state for the Ising model. In section IV we give some conclusions and discussions.

II. LOCAL CONVERTIBILITY OF TWO PURE STATES

In this section, we introduce the notion of local convertibility and give the necessary and sufficient conditions for the local convertibility used in this paper. Generally, local convertibility concerns the following question: given two quantum states, is that possible to convert one to the other merely using local operations (without global operations)? If it is possible, we say there is local convertibility between the states. Otherwise, there is no local convertibility. The answer to this question is related to the comparisons between the entanglements of the two states. A measure of entanglement which does not increase in the process of LOCC besides other basic conditions is defined as entanglement monotone [23]. Entanglement monotone is not unique. Different entanglement monotones reflect certain aspects of the entanglement and could have inequable usages in QIP. In particular, local convertibilities within different kinds of allowed local operations have different entanglement monotones to compose the necessary and sufficient conditions.

The most basic local conversion is LOCC, which is also the most natural restriction in quantum information processing. It has fabulous applications in several fundamental problems in QIP, such as teleportation [19], quantum states discrimination [20], testing the security of hiding bit [21] and quantum channel corrections [22]. By Schmidt decomposition, a bipartite pure quantum state can be written as $|\Psi\rangle_{AB} = \sum_{k=1}^d \sqrt{\lambda_k} |kk\rangle_{AB}$, where d is the rank of the reduced density operator $\rho_{A(B)} = \text{Tr}_{B(A)}(|\Psi\rangle\langle\Psi|)$, and $\{\lambda_k\}_{k=1}^d$ are the Schmidt coefficients in descending order. They satisfy the positive and normalization conditions, i.e., $\lambda_k > 0$ for all $1 \leq k \leq d$, and $\sum_k \lambda_k = 1$. For a given bipartition of a real physical system all the knowledge of the ground state is contained in the Schmidt coefficients [24]. It is pointed out that quantities $\{\sum_{k=l}^d \lambda_k\}_{l=1}^d$ constitute a set of entanglement monotones [25]. For two bipartite pure states $|\Psi\rangle$ and $|\Psi'\rangle = \sum_{k=1}^d \sqrt{\lambda'_k} |kk\rangle_{AB}$, if the majorization relations

$$\sum_{k=l}^d \lambda_k \geq \sum_{k=l}^d \lambda'_k \quad (1)$$

are satisfied for all $1 \leq l \leq d$, state $|\Psi\rangle$ can be converted with 100% probability of success to $|\Psi'\rangle$ by LOCC [26]. Otherwise these two states are incomparable, i.e., neither can state $|\Psi\rangle$ be converted to $|\Psi'\rangle$ nor can state $|\Psi'\rangle$ be converted to $|\Psi\rangle$ by LOCC. Thus, the majorization relations provide a necessary and sufficient condition for the local convertibility with LOCC. In this sense, $\{\sum_{k=l}^d \lambda_k\}_{l=1}^d$ is a minimal and complete set of entanglement monotones for LOCC.

Another useful local conversion which is also the most powerful one is eLOCC, which is also called entanglement catalyst. Entanglement catalyst in LOCC is such a phenomenon that there are some pure states $|\Psi\rangle_{AB}$ and

$|\Psi'\rangle_{AB}$ that they cannot be converted to each other by LOCC alone, because they violate the majorization theorem. However, when the two local parties share certain kind of ancillary entanglement, which is labeled as $|\phi\rangle$, the state with larger entanglement can be converted to the other state by LOCC and the ancillary state does not change after the conversion just like the catalyst in chemistry reactions [27], i.e., $|\Psi \otimes \phi\rangle \rightarrow |\Psi' \otimes \phi\rangle$. For example, $|\Psi\rangle = \sqrt{0.4}|00\rangle + \sqrt{0.4}|11\rangle + \sqrt{0.1}|22\rangle + \sqrt{0.1}|33\rangle$, and $|\Psi'\rangle = \sqrt{0.5}|00\rangle + \sqrt{0.25}|11\rangle + \sqrt{0.25}|22\rangle + \sqrt{0}|33\rangle$. It can be easily checked that $\lambda_2 + \lambda_3 + \lambda_4 > \lambda'_2 + \lambda'_3 + \lambda'_4$, but $\lambda_3 + \lambda_4 < \lambda'_3 + \lambda'_4$, therefore, neither state can be converted to the other with certainty, i.e., $|\Psi\rangle \not\rightarrow |\Psi'\rangle$ and $|\Psi'\rangle \not\rightarrow |\Psi\rangle$. Whereas if $|\phi\rangle = \sqrt{0.6}|44\rangle + \sqrt{0.4}|55\rangle$ is applied, the Schmidt coefficients for the product state $|\Psi\rangle|\phi\rangle$ and $|\Psi'\rangle|\phi\rangle$ in decreasing order are $\Lambda = \{0.24, 0.24, 0.16, 0.16, 0.06, 0.06, 0.004, 0.04\}$ and $\Lambda' = \{0.30, 0.20, 0.15, 0.15, 0.10, 0.10, 0.00, 0.00\}$, such that $\sum_{k=l}^8 \Lambda_k \geq \sum_{k=l}^8 \Lambda'_k, \forall 1 \leq l \leq 8$. As a result, $|\Psi\rangle|\phi\rangle$ can be converted to $|\Psi'\rangle|\phi\rangle$ with certainty by LOCC. We can see that the LOCC with ancillary assisted-entanglement (eLOCC) actually enlarges the previous Hilbert space and is a generalized version of LOCC.

The eLOCC conversion can be determined by the Rényi entropy [28], which is defined as

$$S_\alpha(\rho_A) = \frac{1}{1-\alpha} \log \text{Tr} \rho_A^\alpha. \quad (2)$$

In particular, when $\alpha \rightarrow 1$, it tends to the von Neumann entropy. Rényi entropy was extensively studied in spin chain systems [29]. States $|\Psi\rangle_{AB}$ can be converted to $|\Psi'\rangle_{AB}$ by eLOCC iff their Rényi entropies satisfy $S_\alpha(\rho_A) \geq S_\alpha(\rho'_A)$ for all α [30]. So in the graph of Rényi entropy versus α , if the Rényi entropies of states $|\Psi\rangle_{AB}$ and $|\Psi'\rangle_{AB}$ intercept, these two states are incomparable, see Fig.1 (right). On the other hand, if there is no interception, the state with larger entanglement can be locally converted to the other state, see Fig.1 (left). Therefore, the Rényi entropies are a minimal and complete set of entanglement monotones for eLOCC. In the following, we focus on the local convertibility within eLOCC.

Now we consider the Rényi entropy in quantum critical systems. The wave functions of ground states from different quantum phases are drastically distinct. When quantum phase transition occurs, the properties of the ground state must change abruptly [31]. We argue that as part of this general feature of quantum phase transition, the interception of ground state Rényi entropies as well as the local conversion property of the ground state will change at the critical point, and the different quantum phases boundaries can be determined by the Rényi entropy. By carefully analyzing the behavior of Rényi entropy, we find two possible cases: (i) Please see Table I (left). In phases I, the Rényi entropies of any two different ground states intercept, while in phases II, any two different ground states do not intercept. And the Rényi entropies of two states from different phases will intercept. That means for any two states in phase II

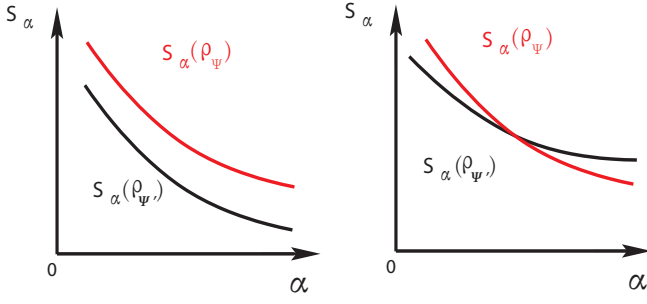


FIG. 1: (Color online). Rényi entropies of two bipartite states $|\Psi\rangle$ and $|\Psi'\rangle$ may have two types of behavior: they intercept or not. For interception case (right), the two states cannot be locally converted to each other. For non-interception case (left), $|\Psi\rangle$ can be locally converted to $|\Psi'\rangle$.

TABLE I: Interceptions of the ground states Rényi entropies, where \times means Rényi entropies are intercepted and N means the non-interception. The left table is for case (i) where the phase boundary can be obtained along the diagonal elements. The right table corresponds to the case (ii) where the phase boundary can be obtained with the help of the anti-diagonal elements.

	phase I	phase II		phase I	phase II
phase I	\times	\times	phase I	N	\times
phase II	\times	N	phase II	\times	N

the one with larger entanglement can be converted to the other one via eLOCC, while in phase I any ground state cannot convert to other states via eLOCC, as a result global operation is necessary in phase I. The corresponding examples for this type are the transverse field Ising model and XY model. (ii) Please see Table I (right). The ground states Rényi entropies do not intercept with others in the same phase, but they intercept in the different phases. That means the ground state can be converted through eLOCC within the same phase. However, the ground state cannot be converted locally into the different phases. The corresponding example is XXZ model. These two scenarios can be used to detect quantum phase transitions.

III. ELOCC IN QUANTUM CRITICAL SYSTEMS

In this section we use the above method to study some typical quantum critical systems. For a Hamiltonian $H(\lambda)$ with a critical point λ_c , we change the parameter λ gradually from phase I to phase II. For each given λ , we calculate and plot the ground state S_α with respect to α . We expect to see the two cases described in Table I emergence.

We first consider the one dimensional spin 1/2 XY

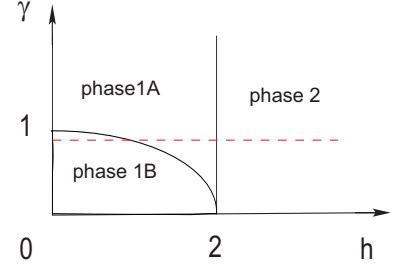


FIG. 2: (Color online). Phase diagram of the XY model. The phase transition from phase 1A or phase 1B to phase 2 referred to as the *Ising* transition is driven by the transverse magnetic field h , and the phase boundary $h = 2$ is a critical line. Phase 1 has two distinct regions A and B. The boundary $\gamma^2 + (h/2)^2 = 1$ is not a critical line. But the properties of this two regions are different. We consider the red dash line $\gamma = \sqrt{3}/2$ in the phase diagram. Thus, it intercepts with the two boundaries at $h = 1$ and $h = 2$, respectively.

chain with the Hamiltonian

$$H = - \sum_{i=1}^N [(1 + \gamma) \sigma_i^x \sigma_{i+1}^x + (1 - \gamma) \sigma_i^y \sigma_{i+1}^y + h \sigma_i^z], \quad (3)$$

where $\sigma_i^{x,y,z}$ stand for the Pauli matrices. γ is coupling parameter and h is field parameter. We can focus on the positive valued γ and h space because of the symmetry of the Hamiltonian. This model and its generalizations have been studied extensively [32]. Fig. 2 shows the phase diagram of this model. This two dimensional phase diagram is a little bit complicated. In order to make a clearer description of the eLOCC proposal, we first employ the transverse field *Ising* model, which is a special case of the XY model to show how this method works. We can obtain the transverse field Ising model from the XY Hamiltonian by setting $\gamma = 1$, $h = 2g$ and removing the unimportant global factor 2 for each components as

$$H_I = - \sum_{i=1}^N (\sigma_i^x \sigma_{i+1}^x + g \sigma_i^z). \quad (4)$$

A second order quantum phase transition takes place at $g = 1$. When $g \neq 1$, there is a energy gap between the ground state and the first excited state, and this gap vanishes when $g = 1$ [31].

We calculate the Rényi entropies of the ground states with the parameter g varying from 0.5 to 1.5 and plot them in Fig. 3. Here the system size $N = 10$ and periodic boundary condition is assumed, i.e., $\sigma_{N+1}^{x,y} = \sigma_1^{x,y}$. To calculate the Rényi entropy we need to know the ground state, which is obtained by diagonalizing the whole Hamiltonian using Matlab. Although the system size which can be calculated is relatively small, the advantage of directly diagonalizing the whole Hamiltonian is the high accuracy. Other numerical methods, such as Lanczos algorithm, DMRG and so on are worth generalizing in this proposal, but as we are conceiving a new

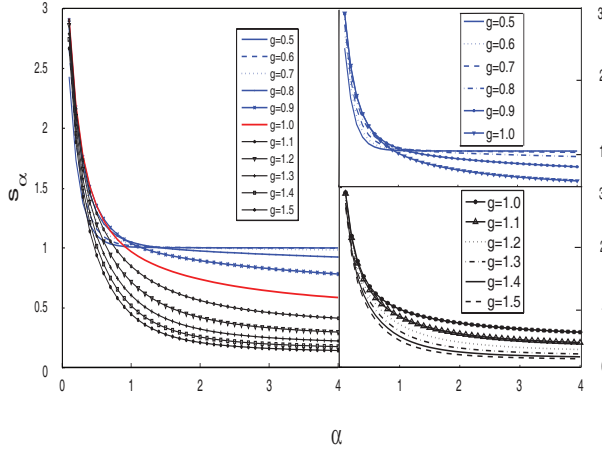


FIG. 3: (Color online). Rényi entropy for the ground state of the transverse field Ising model versus the parameter α . The total site number N is taken to be 10, and the spins are cut into two blocks with 5 sites respectively. Periodical boundary condition is assumed. The blue lines are for the ground states with $g = 0.5, 0.6, 0.7, 0.8, 0.9$, and the black lines are for the ground states with $g = 1.1, 1.2, 1.3, 1.4, 1.5$, respectively. The red line is $g = 1$.

proposal to investigate the quantum critical point rather than developing a different numeric technique within the framework of those already known proposals, we focus on the basic diagonalization method, and the application of other numeric methods are not within the scope of the present paper.

The results have shown that these states can be classified into three distinct groups: In group I (blue line) $g < 1$; In group II (red line) g is at transition point and in group III (black line) $g > 1$. Group I and III are two phases and group II is the phase transition region, which is the boundary of I and III. Fig. 3 clearly shows that in group I Rényi entropies for different ground states intercept with each other. While in group III the Rényi entropy of different states never intercepts with each other. Apart from that the Rényi entropy from different phases will intercept.

These results mean that although in region I and III the model are both gapped their ground states convertibility by eLOCC are quite different: In the ferromagnetic phase where $g < 1$, the ground states cannot convert to each other because their Rényi entropy intercepts; while in the paramagnetic phase where $g > 1$, the ground states can convert by eLOCC because their Rényi entropies never intercept. In addition, states from different regions cannot convert to each other by eLOCC. We can conclude from here that the phase transition has its global signature, and the long range correlations which influence the eLOCC convertibility must play a fundamental role.

Due to the resolving limit of human eyes, we can illustrate the results better by directly comparing the Rényi entropy data in a table instead of the S_α versus α fig-

TABLE II: Interception table of transverse field Ising model near the critical point. For clearance, we cut the table into separate parts, $g \leq 0.98$, $g \geq 1$ and $0.98 \leq g \leq 1$.

g	0.94	0.95	0.96	0.97	0.98	0.99	1.00	1.01	1.02	1.03	1.04
0.94	N	0.6	0.5	0.5	0.5	0.4	0.4	0.3	0.3	0.2	N
0.95	0.6	N	0.5	0.5	0.4	0.4	0.3	0.3	0.2	N	N
0.96	0.5	0.5	N	0.4	0.4	0.3	0.3	0.2	N	N	N
0.97	0.5	0.5	0.4	N	0.3	0.3	0.2	N	N	N	N
0.98	0.5	0.4	0.4	0.3	N	0.2	N	N	N	N	N
0.99	0.4	0.4	0.3	0.3	0.2	N	N	N	N	N	N
1.00	0.4	0.3	0.3	0.2	N	N	N	N	N	N	N
1.01	0.3	0.3	0.2	N	N	N	N	N	N	N	N
1.02	0.3	0.2	N	N	N	N	N	N	N	N	N
1.03	0.2	N	N	N	N	N	N	N	N	N	N
1.04	N	N	N	N	N	N	N	N	N	N	N

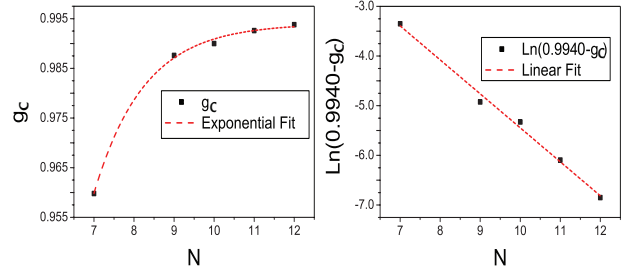


FIG. 4: (Color online). The finite size scaling analysis of Ising model. In the thermodynamic limit, the critical point labeled as g_c obtained by our method tends to 0.9940 with the accuracy of 0.0001. The data can be fitted as $g_c = -9.149e^{-N/1.2522} + 0.9940$.

ure, please see table II. It shows whether any two ground states intercept or not. For example in the second row of Table II, we find that the ground state of $g = 0.94$ intercepts with those of $g = 0.95, 0.96, \dots$, at $\alpha = 0.6, 0.5, \dots$. For the sack of space limit and clarity, we only present the segment of g taking from 0.94 to 1.04. Notice that the diagonal elements are always 'N', which means there is no interception between the corresponding two states, because they stand for two same states must completely overlap but not intercept. By Table II, we find the phase transition lies in $0.98 \leq g \leq 1.00$. We can go on investigating this phase transition more accurately by the same method and we list the result here: when the accuracy (g step) is 0.001 the critical region obtained by this method is $0.987 \leq g \leq 0.989$, and the critical region is $0.9883 \leq g \leq 0.9885$ with the accuracy of 0.0001. We can see from the results that the interval becomes smaller as the varying step of the parameter g becomes more accurate. Moreover, the critical value we obtain is not exactly 1 because we only calculate a finite chain with 10 spins. To get rid of the finite size effect, we give the scaling analysis in Fig.4. When $N \rightarrow \infty$, the phase transition point obtained by this proposal tends to 0.9940. This Ising model corresponds to the first case of the proposal described in the previous section, and Table II corresponds to the left type of Table I.

So far we have shown the role of Rényi entropy in

detecting the critical point by investigating the eLOCC convertibility between different ground states. Then we find the eLOCC convertibility between the ground state and the first excited state also gives good discrimination of different phases. In the ferromagnetic phase where $g < 1$, the Rényi entropy of the ground state and the corresponding first excited state intercepts, while in the paramagnetic phase where $g > 1$, the two Rényi entropies have no interception. We show this in Fig. 5. Moreover the difference of the Rényi entropy in the large α limit between the excited state and the ground state becomes larger with the increasing of the energy gap in the paramagnetic phase.

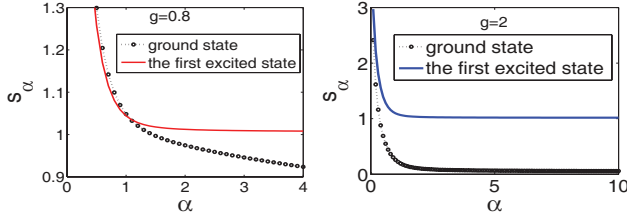


FIG. 5: (Color online). Rényi entropies of ground state and the first excited state. The dash lines are for the ground states, and the solid lines are the first excited states.

TABLE III: Interception table of the XY model near the first boundary $h = 1$.

h	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4	1.5
0.5	N	N	N	N	N	N	N	N	0.3	1.5	1.4
0.6	N	N	N	N	N	N	N	N	0.2	1.5	1.5
0.7	N	N	N	N	N	N	N	0.5	1.6	1.6	1.5
0.8	N	N	N	N	N	N	N	1.6	1.8	1.6	1.5
0.9	N	N	N	N	N	N	1.4	2	1.9	1.7	1.5
1	N	N	N	N	N	N	2.2	2.2	1.9	1.7	1.5
1.1	N	N	N	N	1.4	2.2	N	2.2	1.9	1.7	1.5
1.2	N	N	0.5	1.6	2	2.2	2.2	N	1.8	1.7	1.5
1.3	0.3	0.2	1.6	1.8	1.9	1.9	1.9	1.8	N	1.6	1.5
1.4	1.5	1.5	1.6	1.6	1.7	1.7	1.7	1.7	1.6	N	1.4
1.5	1.4	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.4	N

TABLE IV: Interception table of the XY model near the critical point $h = 2$.

h	1.6	1.7	1.8	1.9	2	2.1	2.2	2.3	2.4	2.5	2.6
1.6	N	1.1	1	0.9	0.8	0.7	0.6	0.5	0.4	0.3	N
1.7	1.1	N	0.9	0.8	0.7	0.6	0.4	0.2	N	N	N
1.8	1	0.9	N	0.7	0.6	0.4	0.3	N	N	N	N
1.9	0.9	0.8	0.7	N	0.4	0.2	N	N	N	N	N
2	0.8	0.7	0.6	0.4	N	N	N	N	N	N	N
2.1	0.7	0.6	0.4	0.2	N	N	N	N	N	N	N
2.2	0.6	0.4	0.3	N	N	N	N	N	N	N	N
2.3	0.5	0.2	N	N	N	N	N	N	N	N	N
2.4	0.4	N	N	N	N	N	N	N	N	N	N
2.5	0.3	N	N	N	N	N	N	N	N	N	N
2.6	N	N	N	N	N	N	N	N	N	N	N

In order to study the general case of XY model we set $\gamma = \sqrt{3}/2$ and varying h from 0 to 3, see the red dash line in Fig.2. We can use the eLOCC proposal to determine the critical point at $h = 2$ as well as the boundary at $h = 1$. Here we just list the results: considering the

eLOCC convertibility between ground states, there is no interception in region 1B, but every two ground states in region 1A intercept, and then in phase2 there is no interception again, please see table III and IV. The boundary at $h = 1$ and the critical point at $h = 2$ also correspond to the first case introduced in the previous section. Table III and IV are corresponding to the left type in Table I. There are interceptions between region 1B and phase2. For the case of total lattice number $N = 10$, we detect the first boundary lies in the range $0.999 \leq h \leq 1.000$ and the critical point is $2.010 \leq h \leq 2.012$ with the accuracy of 0.001.

Next, we study one dimensional XXZ model with the Hamiltonian

$$H_{XXZ} = \sum_i \sigma_i^x \sigma_{i+1}^x + \sigma_i^y \sigma_{i+1}^y + \Delta \sigma_i^z \sigma_{i+1}^z, \quad (5)$$

where Δ is the anisotropic parameter. There are two phase transition points: $\Delta = -1$ corresponds to a first order phase transition, and $\Delta = 1$ is a continuous phase transition of infinite order [33]. In particular, the phase transition at $\Delta = 1$ is a Kosterlitz-Thouless like transition, which can be described by the correlation length divergency but without long range order [34]. We focus on identifying the critical point $\Delta = 1$ by the eLOCC proposal. Here we use the same method as we did in the Ising model and XY model to get the ground state Rényi entropy, i.e., diagonalizing the whole Hamiltonian to obtain all the eigenstates, then we select the ground state to calculate the eigenvalues of reduced density matrix to obtain the Rényi entropy.

Table V shows the interceptions near $\Delta = 1$. We can see that each state in either region $\Delta \geq 1.0$ or $\Delta \leq 1.0$ never intercepts with any of the states in the same region, but intercepts with at least one state from the other region. Therefore this model corresponds to the second case of the proposal introduced in the previous section. Thus, the critical region can be determined as $0.9 \leq \Delta \leq 1.1$. By narrowing the varying step of Δ , this critical point can be detected more accurately. Therefore, the eLOCC proposal also works well for this infinite order phase transition in XXZ spin chain.

We can find that result of XXZ model resembles the right pattern of Table I, i.e., the ground states do not intercept with the states from the same phase, but intercepts the states from the other phase.

IV. CONCLUSIONS

In this paper, we analyzed the Rényi entropy and the eLOCC convertibility in quantum critical systems. We developed a new proposal to describe the eLOCC conversion properties in quantum critical systems by examining the Rényi entropy interception, which shows the disability of eLOCC conversion. We applied this proposal to the study of several typical quantum critical systems, e.g. one dimensional transverse field Ising model, XY

TABLE V: Interception table of XXZ model. For clearance, we cut the table into separate parts, $\Delta \leq 0.9$, $\Delta \geq 1.1$ and $0.9 \leq \Delta \leq 1.1$. It is a 10 sites chain with comb like partition, i.e., the odd numbered sites belong to part A.

Δ	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5	1.6
0.4	N	N	N	N	N	N	N	N	N	N	N	5.4	1.1
0.5	N	N	N	N	N	N	N	N	N	N	N	1	N
0.6	N	N	N	N	N	N	N	N	N	N	1	N	N
0.7	N	N	N	N	N	N	N	N	N	0.9	N	N	N
0.8	N	N	N	N	N	N	N	N	0.9	N	N	N	N
0.9	N	N	N	N	N	N	N	0.9	N	N	N	N	N
1.0	N	N	N	N	N	N	N	N	N	N	N	N	N
1.1	N	N	N	N	N	0.9	N	N	N	N	N	N	N
1.2	N	N	N	N	0.9	N	N	N	N	N	N	N	N
1.3	N	N	N	0.9	N	N	N	N	N	N	N	N	N
1.4	N	N	1	N	N	N	N	N	N	N	N	N	N
1.5	5.4	1	N	N	N	N	N	N	N	N	N	N	N
1.6	1.1	N	N	N	N	N	N	N	N	N	N	N	N

model and XXZ model. The results showed that: the eLOCC convertibility changes at critical points. For the Ising phase transition, eLOCC convertibility in the paramagnetic phase is stronger than that in the ferromagnetic phase in two ways: (i) any two ground states in paramagnetic phase can convert by eLOCC but those in ferromagnetic phase cannot; (ii) each first excited state in paramagnetic phase can be converted to their ground states by eLOCC while states in ferromagnetic phase cannot. For the XY model with two dimensional phase diagram, the critical points can be determined via this method at very high accuracy. The boundary between region 1A and 1B can be detected as well. For the XXZ model the infinite order phase transition at $\Delta = 1$ can also be determined by this method while the pattern of the interception table is different from the second order quantum phase transitions in Ising model and XY model.

In fact, as is shown in Table I, the Rényi entropy in-

ception table can be divided into four blocks. The two anti-diagonal blocks represent the comparison between ground states from two different phases, and it is not surprising that the two blocks are crossings which means that the ground states from the two phases are too different to convert by local operations. The two diagonal blocks represent the comparison between the ground states from the same phase. We find two possible cases in Table I, which are two extremes. Case (i) has the most crossings in the table, while case (ii) has the least crossings. As a matter of fact, The crossings in the table reflect the degree of how hard it is to convert the different ground states, which can be served as a measure of the difference between the two phases. It is quite interesting to find that this observation is amazingly consistent with the orders of the phase transitions in the two example models. In case (i), i.e., the second order quantum phase transition which is the lowest order that quantum critical phenomena can exist, we find the pattern with the most crossings, while in case (ii), i.e., the infinite order quantum phase transition, we find the pattern with the least crossings.

The eLOCC proposal may help further understanding the complicated phenomena of quantum critical systems. This paper would initiate extensive studies of quantum phase transitions from the brand new perspective of local convertibility. This simple but effective method is worth (a) generalizing to study finite temperature phase transitions (b) generalizing based on the majorization scheme [26] and (c) applying to other systems.

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- [1] T. D. Ladd, F. Jelezko, R. Laflamme, Y. Nakamura, C. Monroe and J. L. O'Brien, *Nature*, **464**, 45 (2010).
 - [2] L. Amico, R. Fazio, A. Osterloh, and V. Vedral, *Rev. Mod. Phys.* **80**, 517 (2008).
 - [3] G. Vidal, J. I. Latorre, E. Rico, and A. Kitaev, *Phys. Rev. Lett.* **90**, 227902 (2003).
 - [4] T. J. Osborne, and M. A. Nielsen, *Phys. Rev. A* **66**, 032110 (2002).
 - [5] A. Osterloh, L. Amico, G. Falci and R. Fazio, *Nature* **416**, 608 (2002).
 - [6] L.-A. Wu, M. S. Sarandy, and D. A. Lidar, *Phys. Rev. Lett.* **93**, 250404 (2004);
 - [7] R. Orús and T.-C. Wei, *Phys. Rev. B* **82**, 155120 (2010).
 - [8] R. Dillenschneider, *Phys. Rev. B* **78**, 224413 (2008).
 - [9] J. Cui, J.P. Cao, H. Fan, *Phys. Rev. A*, **82**, 022319 (2010).
 - [10] H. Li and F. D. M. Haldane, *Phys. Rev. Lett.* **101**, 010504 (2008); R. Dillenschneider, *Phys. Rev. B*, **78**, 224413 (2008); H. Yao, and X.L. Qi, *Phys. Rev. Lett.* **105**, 080501 (2010).
 - [11] S. T. Flammia, A. Hamma, T. L. Hughes, and X.G. Wen, *Phys. Rev. Lett.* **103**, 261601 (2009).
 - [12] H. T. Quan, Z. Song, X. F. Liu, P. Zanardi, and C. P. Sun, *Phys. Rev. Lett.* **96**, 140604 (2006); P. Zanardi and N. Paunkovic, *Phys. Rev. E* **74**, 031123 (2006); S.J. Gu, *Int. J. Mod. Phys. B* **24**, 4371 (2010); P. Buonsante and A. Vezzani, *Phys. Rev. Lett.* **98**, 110601 (2007); P. Zanardi, P. Giorda, and M. Cozzini, *Phys. Rev. Lett.* **99**, 100603 (2007); H. Q. Zhou, R. Orus, and G. Vidal, *Phys. Rev. Lett.* **100**, 080601 (2008).
 - [13] W. L. You, Y. W. Li, and S. J. Gu, *Phys. Rev. E* **76**, 022101 (2007).
 - [14] H. T. Quan, F. M. Cucchiatti, *Phys. Rev. E* **79**, 031101 (2009).
 - [15] L. Campos Venuti, C. Degli Esposti Boschi, M. Roncaglia, and A. Scaramucci, *Phys. Rev. A*, **73**, 010303 (R) (2006).
 - [16] A. Anfossi, P. Giorda, and A. Montorsi, *Phys. Rev. B*, **78**, 144519 (2008).
 - [17] W. Dür, G. Vidal and J. I. Cirac, *Phys. Rev. A* **62**, 062314 (2000).
 - [18] C. H. Bennett, H. J. Bernstein, S. Popescu, and B. Schu-

- macher, Phys.Rev.A **53**, 2046 (1996).
- [19] D. Bouwmeester, J. W. Pan, K. Mattle, M. Eibl, H. Weinfurter, A. Zeilinger, Nature, **390**, 575 (1997).
 - [20] L. C. Robert; J. M. Paul and J. M. Geremia, Nature, **446**, 774 (2007).
 - [21] B. M. Terhal, D. P. DiVincenzo, and D. W. Leung, Phys. Rev. Lett. **86**, 5807 (2001).
 - [22] P. Hayden and C. King, Quantum Information and Computation **5** (2), 156 (2005).
 - [23] G. Vidal, J. Mod. Opt. **47**, 355 (2000).
 - [24] D. Poilblanc, arXiv:1011.2147 (2010).
 - [25] G. Vidal, J. Mod. Optics. **47**, 355 (2000); G. Vidal, Phys. Rev. Lett. **83**, 1046 (1999).
 - [26] M. A. Nielsen, Phys. Rev. Lett. **83**, 436 (1999).
 - [27] D. Jonathan and M. B. Plenio, Phys. Rev. Lett. **83**, 3566 (1999).
 - [28] A. Rényi, in Proceedings of the 4th Berkeley Symposium on Mathematical Statistics and Probability (University of California Press, Berkeley, CA, 1961), **1**, 547.
 - [29] J. Cardy and P. Calabrese, J. Stat. Mech. P04023 (2010); P. Calabrese and F.H.L. Essler, J. Stat. Mech. P08029 (2010); P. Calabrese, M. Campostrini, F. Essler and B. Nienhuis, Phys. Rev. Lett. **104**, 095701 (2010); P. Calabrese, J. Cardy and I. Peschel, J. Stat. Mech. P09003 (2010).
 - [30] S. Turgut, J. Phys. A: Math. Theor. **40**, 12185 (2007); M. Klimesh, arXiv:0709.3680 (2007); Y. R. Sanders and G. Gour, Phys. Rev. A **79**, 054302 (2009).
 - [31] S. Sachdev, Quantum Phase Transition (Cambridge University Press, Cambridge, UK, 1999).
 - [32] V. Mukherjee, U. Divakaran, A. Dutta, and D. Sen, Phys. Rev. B **76**, 174303 (2007); S. Garnerone, N. T. Jacobson, S. Haas, and P. Zanardi, Phys. Rev. Lett. **102**, 057205 (2009); F. Franchini, A. R. Its and V. E. Korepin, J. Phys. A: Math. Theor. **41**, 025302 (2008).
 - [33] C. N. Yang and C. P. Yang, Phys. Rev. **150**, 321 (1966); Phys. Rev. **150**, 327 (1966).
 - [34] M.F. Yang, Phys. Rev. B **76**, 180403 (R) (2007).